Geodesic grassfire for computing mixed-dimensional skeletons

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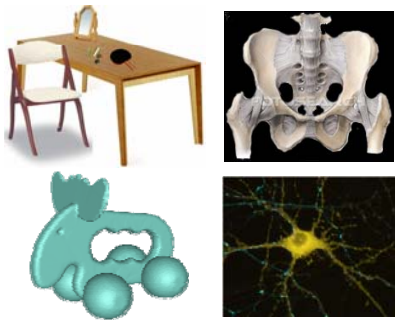
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Introduction

Shape anisotropy

Objects in nature often contain parts that exhibit **anisotropic** elongation, such as tubes (1-D elongation) and plates (2-D elongation). These parts are often strongly associated with the structure, and/or function of the objects.



Goal

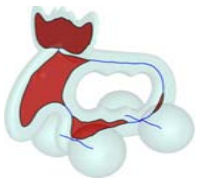
Compute a shape descriptor that captures object parts with different shape anisotropy.

Approach: compute a **mix-dimensional skeleton** that consists of medial elements whose dimension reflects the dimension of anisotropic elongation:

- Medial curve: tube-like parts
- Medial surface: plate-like parts

Applications:

- Recognition
- Segmentation
- Comparison
- Retrieval
- Visualization



Challenges

Existing skeletonization works are **dimension-specific** (e.g., either surface or curve skeleton). Computing a skeleton with elements at multiple dimensions faces two challenges:

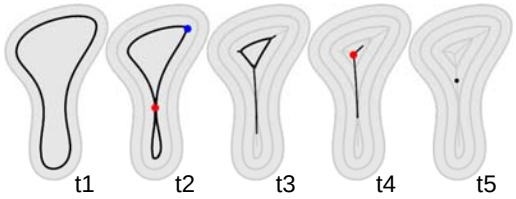
- Defining medial geometry at lower-dimensions (e.g., k-D medial geometry for N-D objects for k<N)
- Identifying salient medial geometry at each dimension (e.g., capturing shape anisotropy at that dimension)

Method

Continuous analogy

Geodesic Grassfire: An erosion that travels geodesically on a manifold from its boundary.

- Lower-dimensional manifolds form as the erosion fronts meet (e.g., dots in **t2**).
- A erosion front ends if the boundary vanishes (e.g., in **t4**)
- Erosion ends when there is no boundary left (e.g., **t5**).



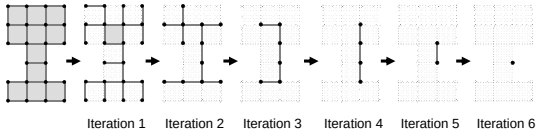
Defining medial geometry: The k-D medial geometry of an N-D object is the set of *k-D manifold* formed during geodesic grassfire. (When k=N-1, it is the Medial Axes)

Medial salience: A point p on a k-D medial geometry is salient, if the erosion on the k-D manifold reaches p *much later* than the erosion on the (k+1)-D manifold. (Intuitively, the erosion time on a k-D manifold expresses the isotropic elongation of shape in k dimensions at p)

Discrete approximation

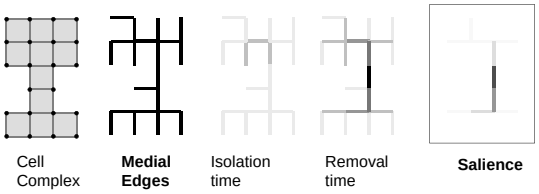
Geodesic Grassfire Thinning: Iterative removal of boundary elements on a cell complex.

- Each iteration simultaneously deletes all pairs of cells on the boundary of the complex.
- Homotopy-preserving



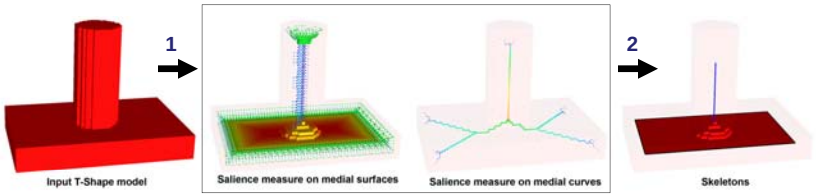
Medial geometry and saliece:

- The k-D medial geometry consists of *isolated* k-D elements during thinning.
- The salience of a medial k-D element is how long it stays *isolated* before it is *removed*.



The algorithm

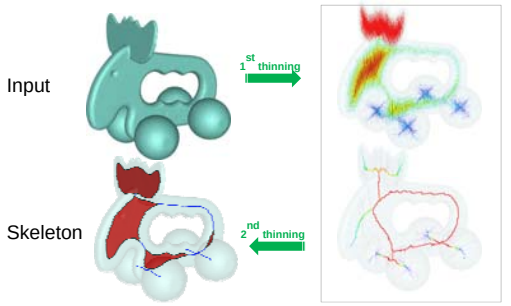
Computes a topology-preserving skeleton that captures shape anisotropy.

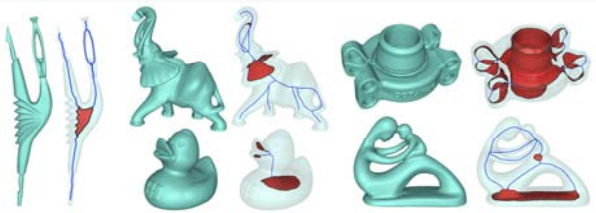


Step 1: Thin the object, compute salience for each k-D medial element.
Step 2: Thin the object again, this time keeping the salience medial elements.

Results

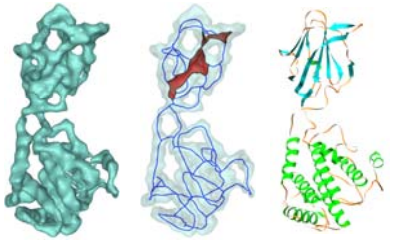
Examples





Name	T-shape	Elephant	Fertility	Protein	Toy	Grayloc	Duck	Dancer
Size(2^n)	6	7	7	7	7	7	7	8
Time(s)	0.67	2.17	2.64	2.66	4.91	6.42	13.94	2.33
Mem(mb)	99.4	158.1	162.4	141.4	244.6	257.0	412.5	161.9

Applications



The mixed-dimensional skeleton computed on a 3D protein model (BTV, left) distinguishes beta-sheets (plate-like, blue on the right) from alpha-helices and loops (tube like).

Conclusion

Contributions

A continuous erosion analogy:

- Define medial geometry in lower-dimensions
- Identify salient medial geometry at each dimension

A discrete algorithm:

- Computes a discrete skeleton that naturally captures shape anisotropy
- Simple to implement
- Generally applicable in any dimensions
- Robust to noise and discretization levels.

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